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BOUNDARY MASS-EXCHANGE CONDITIONS IN THE FORM OF THE NEWTON AND DALTON LAWS

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It has been established that the linear boundary mass-exchange conditions in the form of the Newton law are unsuitable for description of the initial period of drying and the constant-rate period. The nonlinear boundary conditions of the third kind based on the Dalton evaporation law have been proposed. A numerical algorithm for investigation of the temperature and moisture-content fields up to the dropping-rate period has been developed.

Introduction. One formula describing the mass exchange between a moist material and an air medium is the Dalton evaporation law, according to which the intensity of mass exchange is in proportion to the difference of steam partial pressures on both sides of the boundary layer. This expression for the intensity of drying has a clear physical interpretation, and from the viewpoint of A. V. Luikov, it is precisely this expression that is consistent with experiment better than others. Nonetheless, we have found just a small number of works on drying theory, where the Dalton evaporation law is used. Believe that the reason is that this formula represents a *nonlinear* boundary condition with which the initial problem can only be solved by *numerical* methods; at the same time, both A. V. Luikov and his followers mainly adhere to *linearized* heat- and mass-transfer equations and to *analytical* investigation methods based on the Laplace transformation. In such an approach, the boundary mass-exchange conditions are traditionally taken in the form of the Newton linear conditions where the intensity of evaporation of moisture from the specimen surface is in proportion to the difference between the running moisture content on this surface and the equilibrium moisture content of the material. In what follows we will show that, at least for the initial period of drying and the period of constant rate, the Newton conditions result in a conflict with experiment and must be replaced by the Dalton conditions. This fundamental fact is still not clearly understood. We hope that the work proposed will make it possible to eliminate this drawback.

Constant-Rate Period and Mass Exchange by the Newton Law. Let us consider the process of convective drying of a plate manufactured from a moist material (Fig. 1). The condition of heat and moisture insulation of the lower plate surface $x = d$ means that the initial object of study is a specimen of thickness $2d$, in which the heat fluxes and moisture flows through the plate of symmetry $x = d$ are absent due to the identity of the boundary conditions at the boundaries $x = 0$ and $x = 2d$. The problem on finding the fields of the temperature T and the moisture content U will be assumed to be spatially one-dimensional where the functions sought are dependent just on the coordinate x and the time τ, i.e., $T = T(x, \tau)$ and $U = U(x, \tau)$. Such an approximation will be justified if the thickness of the plate d is small compared to its dimensions in directions perpendicular to the x axis, and the intensities of heat and mass exchange of the plate's surface with the incident air flow change along this surface only slightly. The initial boundary-value problem for calculation of the functions T and U has the following form [1]:

$$\frac{\partial T}{\partial \tau} = a \frac{\partial^2 T}{\partial x^2} + \frac{\varepsilon r}{c} \frac{\partial U}{\partial \tau} , \qquad (1)$$

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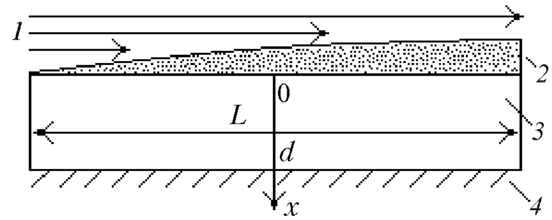

Fig. 1. Scheme of convective drying of a plate manufactured from a moist material: 1) heated-air flow; 2) boundary layer; 3) moist specimen; 4) heat and moisture insulation.

$$\frac{\partial U}{\partial \tau} = a_{\mathrm{m}} \frac{\partial^2 U}{\partial x^2} + a_{\mathrm{m}} \delta \frac{\partial^2 T}{\partial x^2} , \tag{2}$$

$$\alpha [T_{\mathrm{air}} - T(0, \tau)] = -\lambda \frac{\partial T}{\partial x}(0, \tau) + r(1 - \varepsilon) j(\tau) , \tag{3}$$

$$j(\tau) = a_{\mathrm{m}} \rho_0 \left[\frac{\partial U}{\partial x}(0, \tau) + \delta \frac{\partial T}{\partial x}(0, \tau) \right], \tag{4}$$

$$\frac{\partial T}{\partial x}(d, \tau) = 0 , \quad \frac{\partial U}{\partial x}(d, \tau) = 0 , \tag{5}$$

$$T(x, 0) = T_0 , \quad U(x, 0) = U_0 . \tag{6}$$

These relations have not yielded a complete mathematical model of the process so far, since the form of the function $j(\tau)$ calls for additional discussion.

Problems for the diffusion equation are known to possess the property that their solutions at $\tau \to \infty$ cease to be dependent on the initial data. This statement for the particular case has been proved in [2]. Relying on the uniqueness theorem, we can assign the following meaning to it: any solution of the initial boundary-value problem, obtained in one manner or another and satisfying all the conditions formulated minus the initial conditions, will be the solution of the initial problem at $\tau \to \infty$. Such solutions will be called *steady-state* ones. In our case the steady-state solution can be constructed as follows. We set

$$\frac{\partial T}{\partial \tau} = 0 , \quad \frac{\partial U}{\partial \tau} = \mathrm{const} . \tag{7}$$

Since we have $T = T(x)$, it follows from (3) that the intensity of drying $j(\tau)$ is independent of time. Let us introduce the following notation:

$$j(\tau) = \mathrm{const} \equiv j_{\mathrm{w}} , \quad T(x)\Big|_{x=0} \equiv T_{\mathrm{w}} , \quad U(x, \tau)\Big|_{x=0, \tau=\tau_{\mathrm{w}}} \equiv U_{\mathrm{w}} . \tag{8}$$

We show that the steady-state solution of problem (1)–(6) is uniquely determined by conditions (7) and (8). Integrating both sides of (2) for x going from 0 to d, we obtain

$$\frac{\partial U}{\partial \tau} = \frac{1}{d} \left[a_{\mathrm{m}} \frac{\partial U}{\partial x}(x, \tau) + a_{\mathrm{m}} \delta \frac{\partial T}{\partial x}(x, \tau) \right] \Bigg|_{x=0}^{x=d} .$$

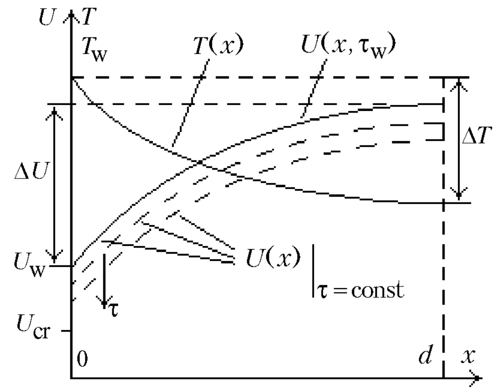

Fig. 2. Temperature and moisture-content distributions in the regime of constant drying rate ($\tau_w < \tau < \tau_{cr}$).

Using boundary condition (4), where it is necessary to set $j(\tau) = j_w$, and boundary condition (5) in substituting the limits of integration, we will have

$$\frac{\partial U}{\partial \tau} = -\frac{j_w}{\rho_0 d} . \tag{9}$$

With account for this formula, Eqs. (1) and (2) are written, after the transformations, as

$$\frac{d^2 T}{dx^2} = -\frac{2\cdot\Delta T}{d^2} , \quad \frac{\partial^2 U}{\partial x^2} = -\frac{2\cdot\Delta U}{d^2} . \tag{10}$$

Here we have introduced the following notation:

$$\Delta T \equiv -\frac{\varepsilon r d j_w}{2\lambda} , \quad \Delta U \equiv -\delta\left(1 + \frac{\lambda}{\varepsilon a_m \rho_0 r \delta}\right)\Delta T . \tag{11}$$

From Eqs. (10), it is seen that the stationary temperature distribution and the simultaneous moisture-content distributions are parabolic. The form of these parabolas is uniquely determined by formulas (5) and (8)–(11). Carrying out the necessary computations, we obtain

$$\frac{T(x) - T_w}{\Delta T} = 1 - \left(\frac{x}{d} - 1\right)^2 , \quad \frac{U(x, \tau) - U_w}{\Delta U} = 1 - \left(\frac{x}{d} - 1\right)^2 - \frac{j_w}{\rho_0 d \Delta U}(\tau - \tau_w) , \quad \tau > \tau_w . \tag{12}$$

The vertices of these parabolas are at the center of the plate (for $x = d$), and the constants ΔT and ΔU have the meaning of temperature and moisture-content differences between the center of the plate and its surface: $\Delta T = T(d) - T(0) < 0$ and $\Delta U = U(d, \tau) - U(0, \tau) > 0$. The form of the parabolic distributions (12) is qualitatively shown in Fig. 2. The quasistationary regime presented in the figure is known to be established indeed. It is called the *regime of constant drying rate*. It begins at a certain instant of time τ_w, when the plate surface temperature becomes equal to the wet-bulb temperature T_w, and ends at the instant τ_{cr}, when the moisture content average over the plate thickness decreases to the critical value U_{cr}. The characteristics of this regime j_w, T_w, U_w, and τ_w are usually determined from experiment. If it is necessary to have a closed mathematical model of the process and to find these parameters from calculations, we must select one law of mass exchange. As has already been noted, in this case one traditionally uses the Newton law

$$j(\tau) = \beta_m [U(0, \tau) - U_{eq}] . \tag{13}$$

However, it is easy to verify that this formula is in conflict with the solution constructed: the quantity $U(0, \tau)$ varies with time (Fig. 2); therefore, contrary to requirement (8), the intensity of drying $j(\tau)$ cannot remain constant. Consequently, it is expedient to consider other mass-exchange models different from the Newton law (13).

Mass Exchange by the Dalton Law. In this case the intensity of drying is determined by the expression

$$j(\tau) = \alpha_m [P_s(\tau) - P_{air}] . \tag{14}$$

Here and in what follows the total pressure of moist air is assumed to be normal. The relative partial pressure of a saturated steam P as a function of its temperature T will be modeled by the formula proposed by G. K. Filonenko [1]:

$$P(T) = 6.03 \cdot 10^{-3} \cdot \exp \frac{17.3 \cdot T}{T_1 + T}, \quad T_1 \equiv 238^{\circ}\text{C} . \tag{15}$$

In what follows, for the sake of brevity, the relative partial pressure of the steam will be called simply pressure. It is well known that, before the period of dropping drying rate, the steam near the material surface is assumed to be saturated. This means that $P_s(\tau) = P(T(0, \tau))$, i.e., the pressure of the steam near the surface is equal to the pressure of a saturated steam at the surface temperature. It follows from determination of the air humidity φ that the steam pressure outside the boundary layer is determined by the formula $P_{air} = \varphi P(T_{air})$. Consequently, the Dalton evaporation law (14) will appear as follows:

$$j(\tau) = \alpha_m [P(T(0, \tau)) - \varphi P(T_{air})] . \tag{16}$$

It is significant that this relation, unlike the Newton formula (13), is consistent with the regularities of the constant-rate period: at a constant surface temperature $T(0, \tau)$, the intensity of drying $j(\tau)$ will remain constant, too, in complete agreement with (8). The coefficients of heat and mass exchange for the laminar state of the boundary layer (Re $< 5 \cdot 10^5$) can be calculated from the formulas [3]

$$\alpha = 0.662 \cdot \frac{\lambda_{air}}{L} \Pr^{1/3} \mathrm{Re}^{1/2}, \quad \alpha_m = 0.662 \cdot \frac{\lambda_m}{L} \Pr_m^{1/3} \mathrm{Re}^{1/2} . \tag{17}$$

Here the parameters λ_{air}, Pr, and $\Pr_m$ (as well as ν in the subsequent discussion) must be determined at the temperature $T_{av} = (T_{air} + T_w)/2$ and humidity $\varphi_{av} = (\varphi + 1)/2$ average over the boundary-layer thickness, and the expression [3, 4]

$$\lambda_m = \frac{D_0 p_0 \mu}{R(T_{av} + T_2)} \left(\frac{T_{av}}{T_2} + 1 \right)^{2.07}, \quad T_2 \equiv 273^{\circ}\text{C} \tag{18}$$

is valid for the moisture conductivity of air. Let us compute the coefficients of heat and mass exchange at $T_{av} = 50^{\circ}$C and $\varphi_{av} = 0.7$. According to the experimental data of [5], here, $\lambda_{air} = 2.73 \cdot 10^{-2}$ W/(m·°C), Pr = 0.73, $\Pr_m$ = 0.58, and $\nu = 1.81 \cdot 10^{-5}$ m²/sec, and calculations from formula (18) yield $\lambda_m = 1.96 \cdot 10^{-5}$ kg/(m·sec). Substituting these values into (17), we obtain

$$\alpha = k_1 \sqrt{\frac{V}{L}}, \quad \alpha_m = k_2 \sqrt{\frac{V}{L}}, \tag{19}$$

where $k_1 \equiv 3.82$ kg/(sec$^{5/2}$·°C) and $k_2 \equiv 2.54 \cdot 10^{-3}$ kg/(sec$^{1/2}$·m²). Let T_{av} and φ_{av} depart from the values taken above in the following limits:

$$20^{\circ}\text{C} < T_{av} < 80^{\circ}\text{C}, \quad 0.6 < \varphi_{av} < 0.8 . \tag{20}$$

An analysis of λ_{air}, λ_m, Pr, $\Pr_m$, and ν as functions of temperature and humidity shows that in this case we must primarily allow for the temperature changes λ_{air}, λ_m, and ν, whereas the remaining changes can be disregarded. Denoting the increments of all the quantities by the symbol Δ and assuming them to be small, from formulas (17) we will have

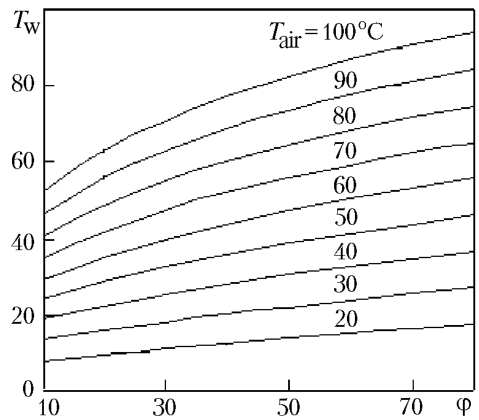

Fig. 3. Wet-bulb temperature T_w vs. temperature of air T_{air} and its humidity φ. T_w, ^oC; φ, %.

TABLE 1. Intensity of Mass Exchange j_w (kg/(m^2·h)) in the Regime of Constant Drying Rate as a Function of the Temperature of Air T_{air} and Its Humidity φ for $V = 2.1$ m/sec and $L = 0.08$ m

T_{air}	φ = 37 %		φ, %	T_{air} = 25°C	
	Calculation	Experiment		Calculation	Experiment
15	0.213	0.22	76	0.100	0.11
25	0.289	0.32	54	0.203	0.18
35	0.365	0.40	37	0.289	0.32
45	0.449	0.50	21	0.382	0.40

$$\frac{\Delta\alpha}{\alpha} \approx \frac{\Delta\lambda_{air}}{\lambda_{air}} - \frac{1}{2}\frac{\Delta\nu}{\nu}, \quad \frac{\Delta\alpha_m}{\alpha_m} \approx \frac{\Delta\lambda_m}{\lambda_m} - \frac{1}{2}\frac{\Delta\nu}{\nu}. \tag{21}$$

From the experimental data of [5], we find that $\Delta\lambda_{air}/\lambda_{air} = 0.075$ and $\Delta\nu = \nu = 0.18$ at $\Delta T = 30^oC$, and computations from formula (18) yield $\Delta\lambda_m/\lambda_m = 0.095$. Substituting these values into (21), we obtain that, when the state of the boundary layer varies in the limits determined by inequalities (20), formulas (19) yield an error no higher than 1.5% for α and 0.5% for α_m. Thus, within the framework of the approximations made, formulas (15), (16), and (19) determine the intensity of drying j as a function of the parameters V, L, φ, and T_{air} and the surface temperature $T(0, \tau)$; the condition of laminar state of the boundary layer has the form $VL < 9.05$ m^2/sec.

Calculation of the Characteristics of the Constant-Rate Period. The constant parameters j_w and T_w are related by the equations

$$j_w = \alpha_m \left[P(T_w) - \varphi P(T_{air})\right], \quad j_w = \frac{\alpha}{r}(T_{air} - T_w). \tag{22}$$

The first relation is a corollary of (16), whereas the second relation is obtained from (3) if we set $T(0, \tau) = T_w$ and $j(\tau) = j_w$ and compute the derivative $\partial T/\partial x(0, \tau)$ using (11) and (12). With account for formulas (19) for α and α_m, from (22) we obtain a transcendental equation for determination of the wet-bulb temperature:

$$P(T_w) - \varphi P(T_{air}) = (T_{air} - T_w)/T_3, \quad T_3 \equiv 1.50 \cdot 10^3\ ^oC.$$

The results (Fig. 3) of numerical solution of this equation are in good agreement with the experimental data of [5]. Relying on the algorithm of calculation of T_w at prescribed T_{air} and φ, we can find, from (22), the intensity of drying j_w as a function of T_{air}, φ, V, and L. From Table 1 it is clear that the results thus obtained differ from the experimental data of [1] by no more than 10%. Thus, for the regime of constant drying rate, the computational algorithm proposed above and based on the Dalton evaporation law leads to a satisfactory agreement with experiment.

Numerical Investigation of Convective Drying. The parameters τ_w and U_w, as well as the temperature and moisture-content distributions at $0 < \tau < \tau_w$, i.e., in the initial period of drying, can only be calculated by numerical methods. For this purpose we introduce a uniform rectangular grid

$$x_j = j\Delta x\,, \quad \Delta x = \frac{d}{n}\,, \quad j = 0, 1, 2, ..., n\,;$$

$$\tau_i = i\Delta\tau\,, \quad \Delta\tau = \frac{\Delta x}{\theta}\,; \quad i = 0, 1, 2, ...\,.$$

The grid functions corresponding to the functions $T(x, \tau)$ and $U(x, \tau)$ sought will be denoted as T_j^i and U_j^i. The array of n T_0^i, T_1^i, ..., T_n^i numbers, which represents the temperature distribution on the ith time layer, will be denoted as T^i. The notation U^i will have an analogous meaning.

Let the arrays T^{i-1} and U^{i-1} be known. We consider the algorithm of calculation of T^i and U^i, i.e., the temperature and moisture-content distributions on a new time layer. First we solve the problem for U^i. Equations (4) and (2) in finite-difference form will become

$$\alpha_m \left[P\,(T_0^{i-1}) - \varphi P\,(T_{air}) \right] = a_m\,\rho_0 \left[\frac{U_1^i - U_0^i}{\Delta x} + \delta\,\frac{T_1^{i-1} - T_0^{i-1}}{\Delta x} \right], \tag{23}$$

$$\frac{U_j^i - U_j^{i-1}}{\Delta\tau} = \frac{a_m}{2\cdot\Delta x^2}\left[U_{j+1}^i + U_{j+1}^{i-1} - 2\,(U_j^i + U_j^{i-1}) + U_{j-1}^i + U_{j-1}^{i-1} \right] + \frac{a_m\delta}{\Delta x^2}\left[T_{j+1}^{i-1} - 2T_j^{i-1} + T_{j-1}^{i-1} \right]. \tag{24}$$

Here the temperature distribution is taken from the previous time layer, i.e., is assumed to be known. In formula (24), the Crank–Nicholson symmetric implicit scheme with a six-point template is used for representation of the second derivative of U, whereas the second derivative of T is approximated by central differences. After the transformations of (23) and (24), with account for the second equation of (5) we will have

$$U_0^i - U_1^i = \delta\left(T_1^{i-1} - T_0^{i-1}\right) - \frac{\alpha_m \Delta x}{a_m\,\rho_0}\left[P\left(T_0^{i-1}\right) - \varphi P\,(T_{air}) \right],$$

$$U_{j-1}^i - 2\left(1 + \frac{\Delta x^2}{a_m \Delta\tau}\right) U_j^i + U_{j+1}^i = -\,U_{j-1}^{i-1} + 2\left(1 - \frac{\Delta x^2}{a_m \Delta\tau}\right) U_j^{i-1} - U_{j+1}^{i-1} -$$

$$-\,2\delta\left(T_{j-1}^{i-1} - 2T_j^{i-1} + T_{j+1}^{i-1}\right), \quad U_{n-1}^i - U_n^i = 0\,, \quad j = 1, 2, ..., n-1\,.$$

This system of linear algebraic equations for unknown U_0^i, U_1^i, ..., U_n^i with a $(n+1) \times (n+1)$ triagonal matrix can be solved by the *marching method* [6], as a result of which the array U^i will be determined. Thereafter the array T^i is found in an analogous manner. Next the procedure described is iterated until the condition $U_{av} < U_{cr}$ is fulfilled. Initial conditions (6) corresponding to the time layer $i = 0$ are those starting for computations.

The parameter of the grid n is determined by the required accuracy of computations, and the minimum possible parameter θ for which the numerical procedure still remains stable is selected. Next we took $n = 50$, and the stability condition had the form $\theta > 20a/d$, or $\Delta x^*/\Delta\tau^* > 20$, where $\Delta x^* = \Delta x/d$ and $\Delta\tau^* = a\Delta\tau/d^2$ are the dimensionless coordinate and time steps of the grid.

For numerical experiments we selected a material with the characteristics of clay: $\lambda = 0.93$ W/(m·^{o}C), $c = 1.8\cdot10^3$ J/(kg·^{o}C), $\rho_0 = 1.5\cdot10^3$ kg/m^3, $a_m = 2.6\cdot10^{-8}$ m^2/sec, $\delta = 1.5\cdot10^{-3}$ 1/^{o}C, $\varepsilon = 0.1$, and $U_{cr} = 0.1$. The characteristics of the air flow outside the boundary layer, the dimensions of the plate, and the initial conditions are as follows: $T_{air} = 100^o$C, $V = 2$ m/sec, $\varphi = 0.5$, $L = 0.4$ m, $d = 0.04$ m, $T_0 = 20^o$C, and $U_0 = 0.4$. We consider the case where a moist plate having a low temperature of $T_0 = 20^o$C begins to be blown with hot air with temperature T_{air} =

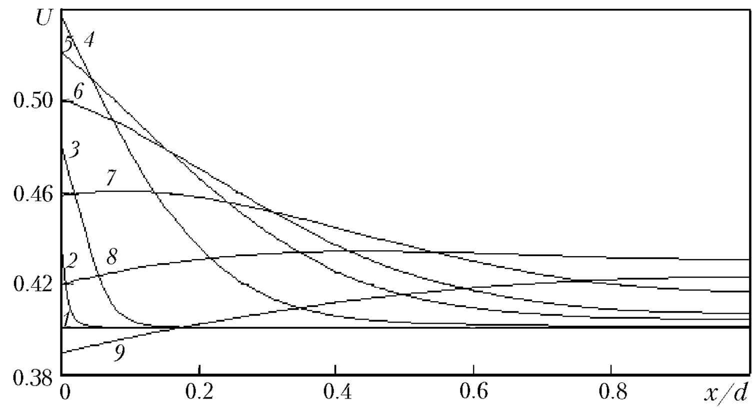

Fig. 4. Distribution of the moisture content U in the initial period of drying: 1) 0, 2) 0.0019; 3) 0.0135, 4) 0.26, 5) 0.7, 6) 1.06, 7) 1.99, 8) 3.87, and 9) 8.39 h. U, kg/kg.

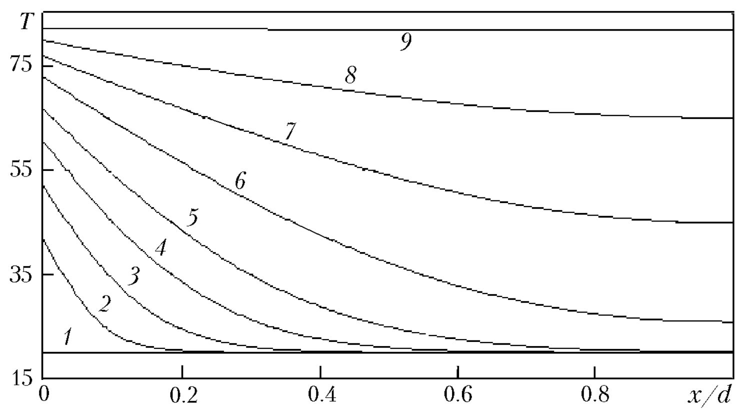

Fig. 5. Distribution of the temperature T before the period of dropping drying rate: 1) 0, 2) 0.01, 3) 0.02, 4) 0.04, 5) 0.07, 6) 0.19, 7) 0.44, 8) 0.88, and 9) 4.0 $< \tau <$ 83.2 h. T, ^{o}C.

100^{o}C at the instant $\tau = 0$. The results of the numerical experiments are presented in Figs. 4 and 5 and in Table 2. It is clear from the table that the condensation of moisture rather than its evaporation occurs on the plate surface before the instant τ = 1.66 h. The reason is that the steam pressure on this time interval turns out to be lower than that in air outside the boundary layer. The condensation of the steam in turn leads to an increase in the moisture content of the material, as is seen in Fig. 4. First the quantity U monotonously increases for all x; the increase is particularly rapid on the surface. Thereafter, nearly from the instant τ = 0.3 h, the moisture content on the surface attains its maximum and begins to decrease, whereas at the center of the plate, it does continue to increase. Finally, at $\tau > 4$ h, U decreases for all x now. Thus, in the initial period of drying, the field of moisture content has the character of a damped wave propagating from the surface of the plate to its center. The amplitude $\Delta U/U_0$ of this wave of increase in the moisture content rapidly decreases, as it moves deep into the specimen: whereas on the surface, it amounts to 35%, at the center, it amounts to only 8%. It is significant that it is precisely such a pattern of development of the moisture-content field that is observed for the stage of heating of the material in experiment [1].

Figure 5 gives an idea of the development of the temperature field, whose feature is a rapid heating of the material throughout the depth at the very beginning of drying. It is significant that this intensification of thermal processes is directly related to the condensation of the steam as discussed above. We consider the manner in which the heat flux $q_{int} = -\lambda \partial T/\partial x(0, \tau)$ coming into the plate from its surface varies with time. According to boundary condition (3), $q_{int} = q_{air} + q_c$, where $q_{air} = \alpha[T_{air} - T(0, \tau)]$ is the heat flux supplied to the surface from air, and $q_c = -r(1 - \varepsilon)j(\tau)$ is the density of the surface heat fluxes due to the condensation of the steam on this surface. It is clear from Table 2 that the condition $q_{int} \approx q_c$, i.e., the initial intense heating of the material occurs mainly due to the heat

TABLE 2. Intensity of Drying and Heat-Flux Densities at Different Instants of Time

τ, h	0	0.103	0.413	0.800	1.66	2.01	2.40	3.02	$4.00 < \tau < 83.2$
j, 10^{-5} kg/(m^2·sec)	–269	–113	–52.4	–23.8	0.0	3.20	5.02	6.23	6.68
q_{air}, W/m^2	683	260	200	175	160	155	151	151	151
q_c, W/m^2	5460	2294	1064	483	0.0	–65	–102	–126	–136
q_{int}, W/m^2	6143	2554	1264	658	160	90	49	25	15

released in steam condensation rather than due to the heat coming from air, is fulfilled nearly to the instant $\tau = 0.4$ h. Nearly from the instant of time $\tau = 4$ h, the numerical solution acts in complete agreement with the analytical solution (12), i.e., the period of constant drying rate begins. The values of the parameters for this period are as follows: $T_w = 82.3^oC$, $j_w = 6.68 \cdot 10^{-5}$ kg/(m^2·sec), $\Delta T = 0.325^oC$, $\Delta U = 0.0342$ kg/kg, $U_w = 0.42$ kg/kg, and $\tau_w = 4.0$ h. The parabolic distributions characteristic of the constant-rate period include curves 9 in Figs. 4 and 5 (the scale selected in the last figure makes it impossible to consider the shape of curve 9, since the temperature difference between the center of the plate and its surface is only 0.3^oC). The constant-rate period ends by the instant $\tau_{cr} = 83.2$ h, when the condition $U_{av} = U_{cr}$ turns out to be fulfilled. The period of dropping rate following the constant-rate period can no longer be considered within the framework of this model, since boundary condition (16) is no longer fulfilled (the steam at the surface ceases to be saturated).

The obtained results of numerical investigation of convective drying with the Dalton boundary conditions are in complete qualitative and quantitative agreement with experimental data. In particular, the numerical experiment has confirmed that, before the dropping-rate period, we have the subdivision of the process into the nonstationary initial period, when transient processes are observed (their duration is nearly 5% of τ_{cr}), and the quasistationary constant-rate period with the characteristic parabolic distributions of the temperature and the moisture content. The solution of this problem with the Newton boundary conditions (it has been obtained in [7]) has no characteristic subdivision into the transient regime and the regime of constant rate and parabolic asymptotics for the moisture-content and temperature fields, and the time variation in these quantities at any instant τ and for any x monotonously obeys a nearly exponential law. As has already been noted, these results are inconsistent with the experimental data for the initial period and the constant-rate period.

Conclusions. A mathematical convective-drying model using the Newton mass-exchange law has been analyzed. It has been shown to be inconsistent with experimental data on the regularities of the initial period of drying and the constant-rate period. Boundary conditions of mass exchange based on the Dalton evaporation law have been proposed for investigation of these periods. The original numerical algorithm for calculation of the wet-bulb temperature and the intensity of drying in the regime of constant rate has been developed. Convective drying has numerically been investigated up to the dropping-rate period by the difference method based on the Crank–Nicholson symmetric implicit scheme with a six-point template. The results of numerical experiments are in good agreement with experimental data. In particular, they confirm A. V. Luikov's qualitative description of the processes of heat and mass transfer in the initial period of drying: if the air temperature is higher than the initial material temperature, the drying begins with the condensation of moisture on the surface rather than from evaporation; the material is rapidly heated due to the heat released, and the moisture begins to move into the specimen, producing a wave of increased moisture content. As far as the Newton mass-exchange conditions are concerned, from our viewpoint, they should be used only for the dropping-rate period; the analytical solution (reported in a number of works) of the problem on convective drying of a plate [7] at least qualitatively follows the regularities of precisely this period.

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NOTATION

$a = \lambda/(c\rho_0)$, coefficient of diffusion of heat, m^2/sec; a_m, coefficient of diffusion of moisture, m^2/sec; c, specific heat of the material, J/(kg·K); d, half the plate thickness, m; $D_0 = 2.05 \cdot 10^{-5}$ m^2/sec, coefficient of diffusion of a steam in air under normal conditions; j and j_w, intensity of mass exchange between the upper boundary of the plate and the air flow and its value in the regime of constant drying rate, kg/(m^2·sec); L, plate length, m; n, parameter determining the coordinate step of the grid; P_s and P_{air}, relative partial pressures of a steam near the plate surface and in air outside the boundary layer; Pr and Pr_m, heat- and mass-exchange Prandtl numbers for air; p_0, normal atmospheric pressure, Pa; q, heat-flux density, W/m^2; r, specific heat of vaporization of water, J/kg; R, universal gas constant, J/(mole·K); Re = VL/ν, Reynolds number; T, T_0, T_{air}, T_w, and T_j^i, temperature of the material, its initial temperature, temperature of air outside the boundary layer, wet-bulb temperature, and grid function, ^{o}C; U, U_0, U_{cr}, U_{av}, U_{eq}, U_w, and U_j^i, moisture content of the material, its initial moisture content, critical moisture content, moisture content average over the plate thickness, equilibrium moisture content, moisture content on the surface at the beginning of the constant-rate period, and grid function, kg/kg; V, velocity of air outside the boundary layer, m/sec; x, Cartesian coordinate, m; α, coefficient of heat exchange of the plate surface with the air flow, W/(m^2·K); α_m and β_m, mass-exchange coefficients of the difference of partial pressure and moisture content, kg/(m^2·sec); δ, coefficient of thermal diffusion of moisture, 1/K; Δx, coordinate step of the grid, m; $\Delta\tau$, time steps of the grid, sec; ε, evaporation coefficient; θ, parameter relating the coordinate and time step of the grid, m/sec; λ, thermal conductivity of the material, W/(m·K); λ_{air}, thermal conductivity of air, W/(m·K); λ_m, moisture conductivity of air, kg/(m·sec); μ, molar mass of water, kg/mole; ν, kinematic viscosity of air, m^2/sec; ρ_0, density of the material in a dry state, kg/m^3; τ, τ_w, and τ_{cr}, running time and instants of the beginning and the end of the period of constant drying rate, sec; φ, humidity of air outside the boundary layer. Subscripts: air, air; int, internal; c, condensation; cr, critical; w, wet bulb; s, surface; eq, equilibrium; av, average; m, moisture.

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